

# Estimating Fatigue Blunting Employing Stress Intensity Fields

Tech Brief 190401 F



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## Abstract:

Blunting of stress concentrations in fatigue behavior is a universally recognized phenomenon. The actual causal mechanism, however, is not fully understood. Although several detailed explanations have been posited as to causation, it is generally accepted that the stress gradient associated with the concentration feature is the primary variable controlling the blunting characterization.

The most common methodology for characterizing fatigue blunting is employing a notch sensitivity factor used to estimate an effective stress concentration which is then used in the fatigue calculation. This tech brief provides a means of using the stress intensity field associated with a notch to predict fatigue behavior. The method provides a coherent approach that accounts for both the geometric and material effects governing the phenomenon. The approach can be used to provide fatigue predictions and to cross check notch sensitivity estimations obtained from other methods.

## Background:

### Peterson's Blunting Model:

Peterson's blunting constant is a typical method for estimating notch sensitivity in a stress concentration. The blunting constant,  $\alpha$ , is a function of the alloy's engineering ultimate strength and the notch sensitivity is a function of the ratio of the blunting constant to the actual notch radius.

$$\alpha = 0.001 \left( \frac{300 \text{ksi}}{\sigma_u} \right)^{1.8}$$

Equation 1

The Peterson notch sensitivity model is given in Equation 2, where  $\rho$  is the notch radius:

$$q = \frac{1}{1 + \frac{\alpha}{\rho}}$$

Equation 2

The effective stress concentration is then computed employing the notch sensitivity factor as shown in equation 3.

$$k_f = (k_t - 1)q + 1$$

Equation 3

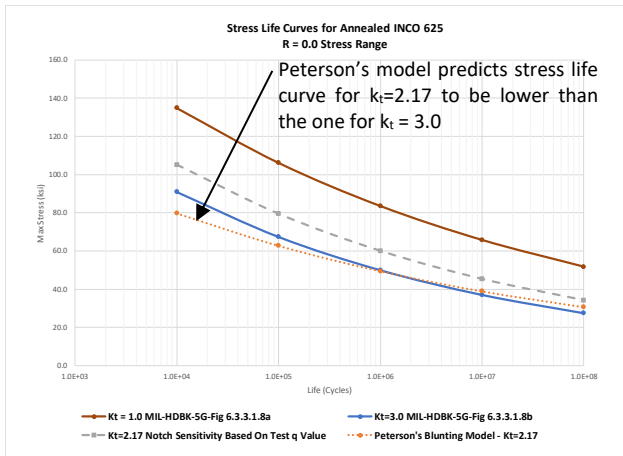
Several potential weaknesses are present in the Peterson blunting model. First, the blunting constant is solely a function of ultimate strength. The ductility which the blunting constant attempts to capture, however, can vary significantly between alloys with similar ultimate strength values. Secondly, notch sensitivity is independent of stress levels. Actual fatigue data, however, demonstrates that this is not the case. Thirdly, the  $k_t$  value is only related to the notch radius. Actual stress concentrations, however, are created by a disruption of the stress field within a structural load path. The effective stress concentration is therefore not only a function of notch size but also dependent on how the load is locally redistributed in the near field.

### Correlation of Peterson Model to Test Data:

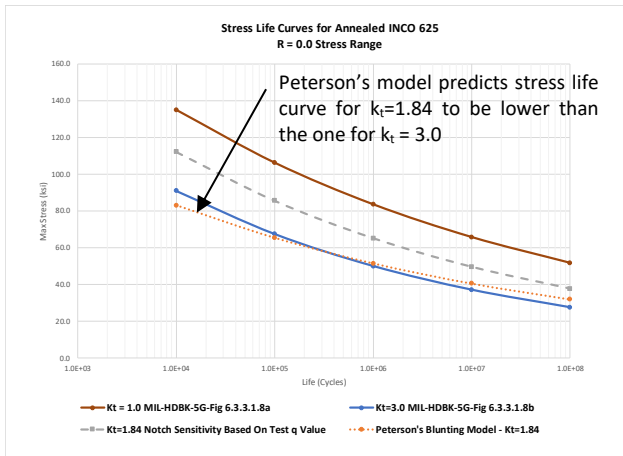
The stress life curves in Figures 1 and 2 illustrate the potential shortfalls of Peterson's blunting model. The curves are typical lives for annealed INCO 625 bar at room temperature.

The source is MIL-HDBK-5G and the empirical data is load controlled for a  $K_t$  of 1.0 and 3.0. The stress range is the nominal net section stress. The test specimens are 0.375 inches in diameter with the  $K_t$  of 3.0 created by a notch which reduces the net section diameter to 0.25 inches. The notch has an included angle of 60 degrees with a radius of 0.013 inches.

The ultimate strength of the alloy is 133 ksi providing a Peterson's blunting constant of 0.0043 inches. Figures 1 and 2 provide the MIL-HDBK stress life curves along with stress curves for stepped shafts with a  $K_t$  of 2.17 and 1.84. The dashed curves are stress life relationships developed from the blunting behavior contained in the actual test data.



**Figure 1 –  $k_t = 2.17$  – Peterson Model**

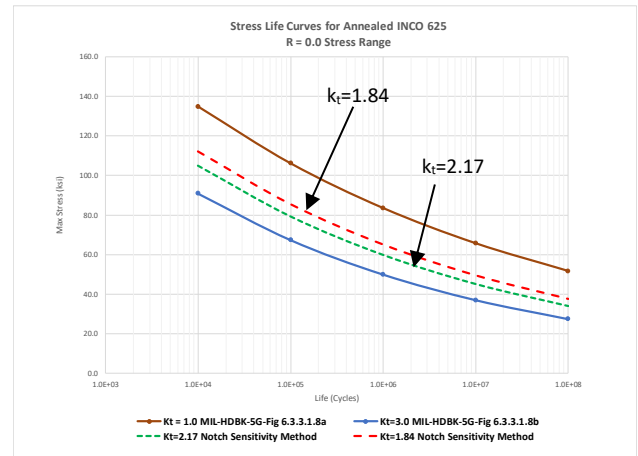


**Figure 2 –  $k_t = 1.84$  – Peterson Model**

In both cases,  $k_t = 2.17$  and  $k_t = 1.84$ , the Peterson predictions provide conservative results. Conservative to the extent, however, that the model provides little actual predictive or explanatory power. An inherent characteristic of conservative methodologies is a loss of contrast between design alternatives. Contrast, however, is necessary to assign proper value to design decisions. The information required for design optimization, root cause failure investigation, or the efficient modification of existing hardware is not available when distinctions in actual performance benefits have been masked.

In this case, Peterson's model cannot make a significant distinction between employing a feature with a  $k_t$  of 2.17 compared to one with a  $k_t$  of 1.84. The notch sensitivity obtained from the test data indicates otherwise. When the  $q$  factor is extracted from the  $k_t$  of 3.0 test data and applied to features

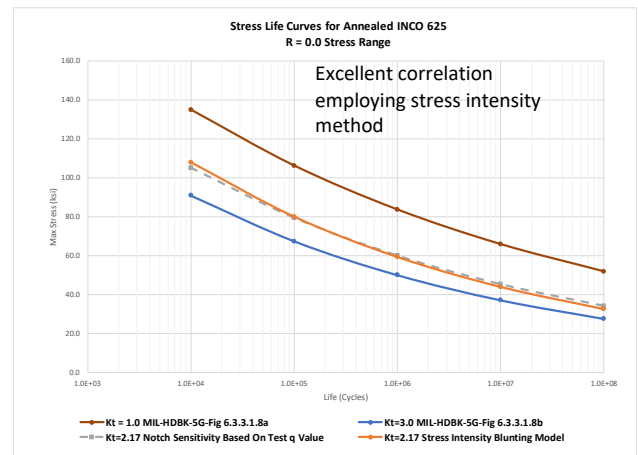
with a  $k_t$  of 1.84 and 2.17 a significant benefit can be seen in using the lower  $k_t$  feature as shown in Figure 3.



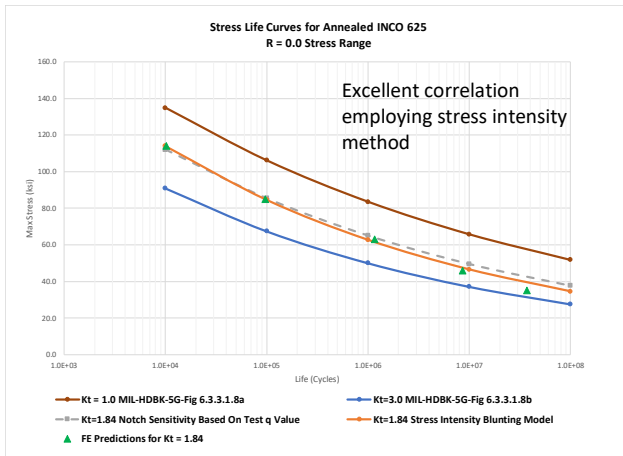
**Figure 3**

**Correlation of Stress Intensity Method to Test Data:**

In contrast to Peterson's model, the stress intensity method provides excellent correlation to the blunting characterization obtained from the test data for both stress concentration values across the entire range from short to long lives. The results of this method for a  $K_t = 2.17$  are shown in Figure 4. To provide additional warrant for the blunting characterization at  $k_t$  values other than 3.0, life predictions based on the notch strain ranges obtained from a finite element (FE) model are provided in Figure 5. The FE results are provided for a  $k_t$  of 1.84 employing kinematic hardening plasticity consistent with a universal slopes Coffin-Manson strain life model. The Morrow mean stress model is used to account for the effect of  $R = 0$  loading.



**Figure 4 –  $K_t = 2.17$  – Stress Intensity Method**



**Figure 5 –  $K_t = 1.84$  – Stress Intensity Method**

The stress intensity method provides a significantly better correlation compared to Peterson’s model. The reasons that this would be expected is that the stress intensity approach provides a more coherent means of accounting for the actual ductility present in the notch and also the effects of load redistribution within the load path. With the examples being stepped shafts, the load redistribution within the process zone is virtually the same as the test specimen and hence the correlation observed. This methodology, however, can also be used to determine the process zone for generalized load path geometry. It provides an excellent means of cross checking fatigue predictions obtained from other methods as well as providing a warrant for assessing whether or not a design is specification compliant.

**Features of the Stress Intensity Method:**

The stress intensity method accounts for the blunting effects on fatigue life by correlating the stress intensity field developed in fatigue notch test specimens to actual load paths incorporating generalized stress concentrations. The stress intensity range captures the effects of both the notch ductility and load redistribution within the near field stress gradient. The stress intensity range, as a function of life cycles, is treated as an invariant.

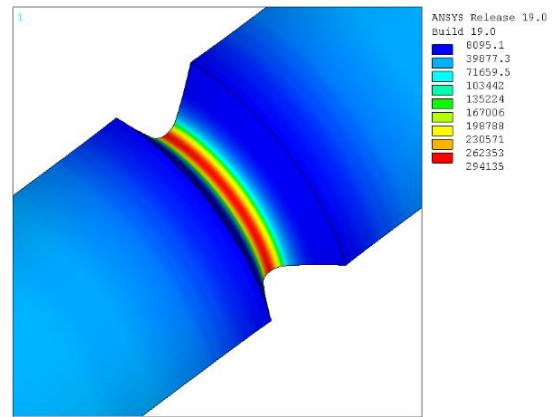
**1. Correlating the Stress Intensity Range to Life Cycle Data:**

The life cycle data from notched test specimen is correlated to the notch stress intensity field using the relationship in equation 4.

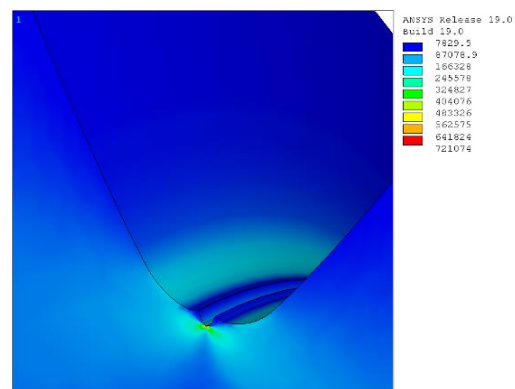
$$k_t = 1 + C_0 \frac{\Delta K_N}{\Delta S_N \sqrt{\pi \rho}}$$

**Equation 4**

Equation 4 is developed from Inglis’ work on stress concentrations of elliptical holes in flat plates and the relationship of the stress intensity field to a far field stress as a function of crack front curvature  $\rho$ . The constant  $C_0$  is obtained from the method discussed in Tech Brief 120701F [Estimating Stress Concentrations with a Minimal Mesh Density Approach](#). Figures 6 and 7 are the test specimen models used to find  $C_0$ .



**Figure 6 – Test Specimen Model**



**Figure 7 – Crack Front Probe to Generate  $C_0$**

Using equation 4, the stress intensity range is correlated to fatigue life as shown in Figure 8.

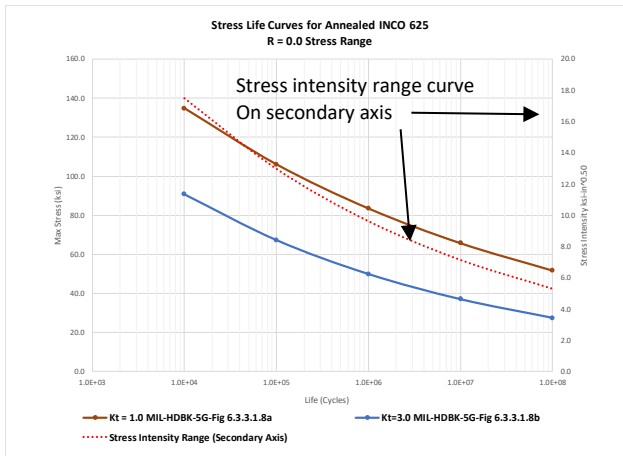


Figure 8

The stress intensity field captures the contribution of both the ductility and load redistribution in the near field of the notch where the fatigue damage is accumulated.

**2. Determine  $C_0$  For the Specific Feature of Interest**

Next, the actual design under evaluation is then modeled and  $C_0$  for equation 4 is determined by the same process as the test specimen. The stepped shaft in Figure 9 is used in illustrating the methodology.

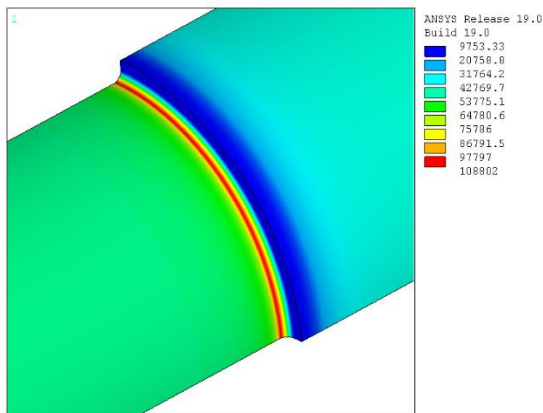


Figure 9 – Stepped Shaft

The constant  $C_0$ , for the stepped shaft, is determined by probing the stress intensity field near the notch as shown in Figure 10 using the method outlined in Tech Brief 120701F.

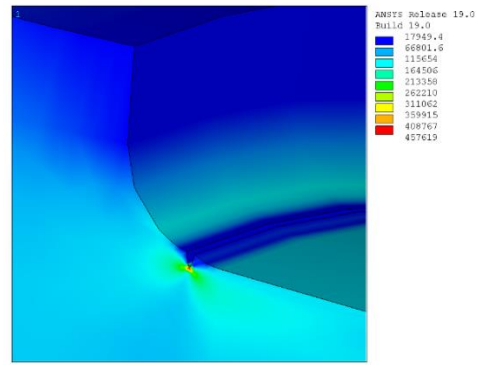


Figure 10 – Crack Front Probe to Generate  $C_0$

**3. Determine Size of the Process Zone and Employ it as the Effective Notch Radius**

The process zone around the stress concentration is where load redistribution will occur and is determined by identifying the distance away from the notch where the stress gradient is present. This zone is dependent on the radius of curvature of the notch, the type of load transfer through the net section (direct or bending), and the geometry. It creates an effective radius through which the load is redistributed as the material undergoes yielding in the process zone.

In the case of the stepped shaft, the load gradient rather than the stress gradient facilitates determining the size of the process zone. This is due to the area of the net section changing as the zone moves away from the notch towards the centerline. The integration of stress to load is not necessary where the area in the net section does not change as a function of distance from the stress concentration.

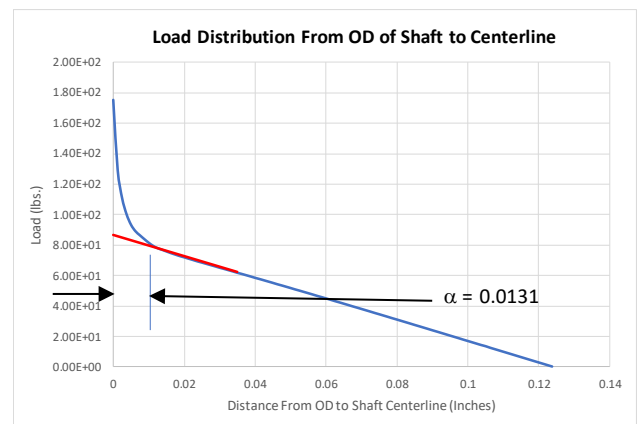


Figure 11 – Process Zone

#### 4. Generate Stress Life Curve for Specific Notch Feature

Using the stress intensity characterization from the notch specimen test data, the correlation constant  $C_0$  for the feature of interest and the associated process zone, equation 5 is employed to generate the stress life curve for the stress concentration in question. In the stepped shaft examples, Figure 4 provides a graph of the stress life curve for the  $k_t = 2.17$  and Figure 5 for a  $k_t = 1.84$ .

$$\Delta S_N = \frac{C_0 \Delta K_N}{(k_t - 1) \sqrt{\pi(\rho + \alpha)}}$$

**Equation 5**

#### ***Conclusion:***

The stress intensity field method provides a coherent means of addressing the primary variables which influence blunting; the size of the process zone, and the alloy's ability to redistribute load within that zone. It provides sufficient predictive and explanatory power which makes cost effective design decisions possible with a high level of confidence in fatigue performance.

As with any fatigue methodology, good practice dictates employing various approaches to assess the sensitivity of predictions associated with the assumptions underpinning each tool. The stress intensity field method should be considered as a viable approach for both assessing and cross checking fatigue blunting estimates for both LCF and HCF predictions in components with complex load paths and transition features.